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The Dynamics of Very Special Black Holes

J. Gutowski

DAMTP, Silver Street, University of Cambridge,
Silver Street, Cambridge CB3 9EW

and

G. Papadopoulos

Department of Mathematics, King's College London,
Strand, London WC2R 2LS

ABSTRACT

We show that the moduli space of supersymmetric black holes that arise in the five-dimensional $N = 2$ supergravity theory with any number of vector multiplets is a weak HKT manifold. The moduli metric is expressed in terms of a HKT potential which is determined by the associated very special geometry of the supergravity theory. As an example, we give explicitly the black hole moduli metric for the STU model.

1. Introduction

In the last few years, there has been much interest in supersymmetric black holes in five dimensions. This is because of the Strominger and Vafa [1] microscopic derivation of the Bekenstein-Hawking entropy for a class of such black holes. The low energy description of M-theory compactifications on six-dimensional Calabi-Yau manifolds [2, 3] and heterotic string theory compactifications on $K_3 \times S^1$ [4] is given by $N = 2$ supersymmetric five-dimensional supergravity theories (eight supercharges) coupled to vector and scalar multiplets [5]. A novel property of the $N = 2$ supergravity theories, as shown by B. de Wit and A. van Proeyen in [6], is that the couplings of the vector multiplets can be determined in terms of geometric data on very special manifolds. These theories admit electrically charged supersymmetric black hole solutions which preserve 1/2 of supersymmetry. Sabra and Chamseddine in [7] have shown that these black holes can be described in terms of the underlying very special geometry.

Viewing the supersymmetric black holes as the BPS soliton solutions of supergravity theories, their low energy dynamics can be approximated by geodesic motion in a moduli space. Some black hole properties like low energy scattering and the presence of bound states can be investigated by studying the geometric structure of the moduli spaces. In turn these may have applications in the understanding of M-theory and its compactifications and perhaps provide another way of deriving the Bekenstein-Hawking entropy formula. Recently it has been realized that the black hole moduli spaces exhibit geometric structures that are related to those that appear on the target spaces of one-dimensional supersymmetric sigma models [8]. This is because the low energy effective theory of black holes, which is an one-dimensional sigma model, has as many supersymmetries as thosed preserved by the associated solutions [9, 10]. An example of such geometry is that of hyper-Kähler with torsion (HKT) [11] which arises in a class of one-dimensional sigma models with four supersymmetries based on the $N = 4B$ multiplet; for applications in mathematics see Grantcharov and Poon [12]. In particular, it has been shown in [10] that the moduli space of five-dimensional black holes that preserve 1/4 of the maximal supersymmetry is a strong HKT manifold. Michelson and Strominger in [13] extended this to five-dimensional black holes which preserve 1/8 of supersymmetry and are electrically charged with respect to the gauge vector potential of the supergravity multiplet. In particular, they established that the moduli space of these black holes admits a weak HKT structure.

In this paper, we investigate the moduli space of electically charged black holes of five-dimensional $N = 2$ supergravity with any number of vector multiplets. We show that the moduli space is a weak HKT manifold. This is in agreement with the counting of the number of unbroken supersymmetries of the black hole solutions and the expected $N = 4B$ multiplet structure of the effective theory. The HKT metric can be expressed in terms of a HKT potential

$$\mu = \int d^4x \, e^{6U} \tag{1.1}$$

where U is determined by the very special geometry of the supergravity theory. In this way, we establish a relation between the very special geometry that arises in $N = 2$ supergravity theory and the weak HKT geometry that arises in one-dimensional supersymmetric sigma models. We give as an explicit example the moduli space metric of the black holes that arise in STU model associated with the $K_3 \times S^1$ compactification of the heterotic string.

This paper is organised as follows: In section two, we describe the supersymmetric electrically charged black hole solutions of $N = 2$ five-dimensional supergravity theory and establish our notation. In section three, we give the black hole moduli metric and show that it is weak HKT. In section four, we present an example associated with the STU model and in section five we give our conclusions.

2. The Black Holes of Five-Dimensional Supergravity

In this section we shall review some facts about very special geometries and their applications to five-dimensional supergravity. The bosonic part action of five-dimensional $N = 2$ supergravity with k vector multiplets is associated to a hypersurface N of $\mathbb{R}^k$ defined by the equation

$$V(X) \equiv \frac{1}{6} C_{IJK} X^I X^J X^K = 1 \tag{2.1}$$

where $\{X^I; I = 1, \ldots, k\}$ are standard coordinates on $\mathbb{R}^k$ and C_{IJK} are constants. In the case of a model arising from a Calabi-Yau compactification of M-theory, the constants C_{IJK} are the topological intersection numbers of the compact manifold. Next we set

$$\begin{aligned} Q_{IJ} &\equiv -\frac{1}{2} \frac{\partial}{\partial X^I} \frac{\partial}{\partial X^J} \log V \mid_{V=1} = \frac{9}{2} X_I X_J - \frac{1}{2} C_{IJK} X^K \\ h_{ab} &= Q_{IJ} \frac{\partial X^I}{\partial \phi^a} \frac{\partial X^J}{\partial \phi^b} \mid_{V=1} , \end{aligned} \tag{2.2}$$

where $\{\phi^a; i = 1, \ldots, k-1\}$ are local coordinates of N, h is interpreted as a metric on N and

$$X_I = \frac{1}{6} C_{IJK} X^J X^K \tag{2.3}$$

are the dual coordinates to X^I. Note that the hypersurface equation $V = 1$ can also be rewritten as $X^I X_I = 1$. Then, the bosonic part of the associated supergravity action [6] with vector potentials A^I and scalars ϕ^a is

$$\begin{aligned} S = \int d^5x \sqrt{-g} \big[R + \frac{1}{2} Q_{IJ} F^I{}_{\mu\nu} F^{J\mu\nu} + h_{ab} \partial_\mu \phi^a \partial^\mu \phi^b \big] \\ - \frac{1}{24} e^{\mu\nu\rho\sigma\tau} C_{IJK} F^I{}_{\mu\nu} F^J{}_{\rho\sigma} A^K{}_\tau \end{aligned} \tag{2.4}$$

where $F^I = dA^I$, $I, J, K = 1, \ldots, k$ are the 2-form Maxwell field strengths, $\mu, \nu, \rho, \sigma = 0, \ldots, 4$, and g is the metric of the five-dimensional spacetime; we have used the same symbol ϕ^a to denote both the coordinates of N and the various scalar fields of the theory.

The field equations of the above Lagrangian obtained from varying the scalars ϕ^a, the spacetime metric g, and the vector potentials A^I are

$$\sqrt{-g}\partial_a Q_{IJ}\big[\frac{1}{2}F^I{}_{\mu\nu}F^{J\mu\nu}+\partial_\mu X^I\partial^\mu X^J\big]-2\partial_\mu\big(\sqrt{-g}Q_{IJ}\partial^\mu X^I\big)\partial_a X^K=0\ , \tag{2.5}$$

$$\begin{aligned}&\sqrt{-g}\big(G_{\mu\nu}+Q_{IJ}F^I{}_{\mu\rho}F^J{}_\nu{}^\rho+Q_{IJ}\partial_\mu X^I\partial_\nu X^J\big)\\ -&\frac{1}{2}\sqrt{-g}g_{\mu\nu}\big[\frac{1}{2}Q_{IJ}F^I{}_{\rho\sigma}F^{J\rho\sigma}+Q_{IJ}\partial_\rho X^I\partial^\rho X^J\big]=0\ ,\end{aligned} \tag{2.6}$$

and

$$-2\partial_\mu\big[\sqrt{-g}Q_{IJ}F^{J\mu\nu}\big]-\frac{1}{8}e^{\nu\rho\sigma\mu\tau}C_{IJK}F^J{}_{\rho\sigma}F^K{}_{\mu\tau}=0\ , \tag{2.7}$$

respectively. The electrically charged black hole solutions [7] that preserve 1/2 of supersymmetry of $N=2$ supergravity action (2.4) are

$$\begin{aligned}ds^2&=-e^{-4U}dt^2+e^{2U}d\mathbf{x}^2\\ A^I{}_0&=e^{-2U}X^I\\ e^{2U}X_I&=\frac{1}{3}H_I\ ,\end{aligned} \tag{2.8}$$

where

$$H_I=h_I+\sum_{A=1}^{N_I}\frac{\lambda_{IA}}{|\mathbf{x}-\mathbf{y}_{IA}|^2} \tag{2.9}$$

is a harmonic function on $\mathbb{R}^4$ with N_I centres. Viewing e^U as an additional scalar, the last equation in (2.8) gives the k independent scalars $\{e^U,\phi^a\}$ in terms of the k harmonic functions $\{H_I\}$. The source term associated to the solution (2.8) is

$$S_{source}=2V_3\int\sum_{I,A}d\tau_{IA}\delta(\mathbf{x}-\mathbf{y}_{IA})\big(X^I\lambda_{IA}-A^I{}_\mu\lambda_{IA}\frac{dy_{IA}{}^\mu}{d\tau_{IA}}\big)\ , \tag{2.10}$$

where V_3 is the volume of the unit three sphere. The addition of (2.10) is due to the presence of non-vanishing delta function sources as $|\mathbf{x}-\mathbf{y}_{IA}|\to 0$.

3. The moduli metric

The moduli space of the black holes (2.8) reviewed in the previous section is expected to be a weak HKT manifold. This is because that although these solutions (2.8) include those that have been used in [13] as a special case, in both cases the killing spinor equations impose to the same conditions on the killing spinors. Consequently, in both cases the sigma models that prescribe the low energy dynamics have the same type of multiplet. The relevant multiplet in this case is the $N = 4B$ which is associated with the weak HKT geometry [8, 10]. In many moduli problems, much of the geometric structure on the moduli space is induced from the geometric structure of the underlying space(time). For the black holes (2.8), the spatial transverse space is $\mathbb{R}^4$ and so it admits a (constant) hypercomplex structure. The hypercomplex structure of the HKT moduli space is induced from that of $\mathbb{R}^4$.

To compute the metric on the black hole moduli space, we follow [14, 15, 16] and allow the positions $\mathbf{y}_{IA}$ to depend on time, i.e.

$$\mathbf{y}_{IA} \to \mathbf{y}_{IA}(t) \ . \tag{3.1}$$

In addition, we perturb the metric and the gauge potentials as

$$\begin{aligned} ds^2 &\to ds^2 + 2e^{-4U} p_m dt dx^m \\ A^I_0 dt &\to A^I_0 dt + (D^I{}_m - e^{-2U} X^I p_m) dx^m \end{aligned} \tag{3.2}$$

where we take p_m and $D^I{}_m$ to be first order in the velocities and we have set $x^\mu = (t, x^m)$, $m, n = 1, \ldots, 4$. The scalar fields are not perturbed linear in the velocities. The p_m and $D^I{}_m$ will be determined by solving the supergravity field equations.

Next we solve the supergravity equations taking into account the source terms to first order in the velocities. The relevant equations are those of the gauge vector potential and those of the metric. The field equation of the scalars vanishes to the first order in the velocities. For convenience, we define

$$
\begin{aligned}
f_{mn} &= \partial_m p_n - \partial_n p_m \\
f^I_{mn} &= \partial_m D^I_n - \partial_n D^I_m \ .
\end{aligned}
\tag{3.3}
$$

Substituting the ansatz (3.2) into the field equations and collecting the terms linear in the velocities, we find

$$
\begin{aligned}
&\partial_0 \partial^n H_I + 3\partial_m \big(e^{-4U} X_I f^{mn} \big) \\
&-2\partial_m \big(e^{-2U} Q_{IJ} f^{Jmn} \big) - \frac{1}{2}\epsilon^{nr\ell m} \partial_r \big(C_{IJK} e^{-2U} X^K \big) f^J{}_{\ell m}) \\
&\qquad + \frac{3}{2}\epsilon^{nr\ell m} \partial_r \big(e^{-4U} X_I \big) f_{\ell m} = 2V_3 \sum_A \lambda_{IA} \delta(\mathbf{x} - \mathbf{y}_{IA}) v_{IA}{}^n
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
e^{2U} \Big[&\frac{1}{2} e^{-2U} X^I \partial_0 \partial_n H_I + \frac{1}{2} e^{-6U} \partial^r (H_J) f^J{}_{nr} \\
&+ \frac{1}{2} e^{-12U} \partial^m \big(e^{6U} f_{mn} \big) \Big] = V_3 \sum_{I,A} X^I \lambda_{IA} \delta(\mathbf{x} - \mathbf{y}_{IA}) v_{IA\ n} \ ,
\end{aligned}
\tag{3.5}
$$

where indices are raised and lowered with respect to the Euclidean metric on $\mathbb{R}^4$. To derive the above equations, we remark that from the field equations of the metric and those of the gauge field only the $0n$ and the n components contribute, respectively. We have also used the identities

$$
\partial_0 \partial_n (e^{2U}) = \frac{1}{3} X^I \partial_0 \partial_n H_I + e^{2U} \partial_n X_I \partial_0 X^I \ . \tag{3.6}
$$

Next we contract the field equation (3.4) with $\frac{1}{2}X^I$ and subtract it from (3.5). This gives

$$
\begin{aligned}
&\partial_m \Big[e^{-6U} \big(f^{mn} - \frac{1}{2}\epsilon^{mnr\ell} f_{r\ell} \big) \\
&- \frac{3}{2} X_I e^{-4U} \big(f^{Imn} - \frac{1}{2}\epsilon^{mnr\ell} f^I_{r\ell} \big) \Big] = 0 \ .
\end{aligned}
\tag{3.7}
$$

Solving this equation for p, we find

$$p^n = -\frac{1}{2V_3}\int d^4z \frac{1}{|\mathbf{x}-\mathbf{z}|^2}\partial_r\big[\frac{3}{2}e^{2U}X_I\big(f^{Irn} - \frac{1}{2}\epsilon^{rn\ell s}f^I_{\ell s}\big)\big] \ . \tag{3.8}$$

Substituting the expression for p back into (3.4), we find

$$D^{Jn} = -\frac{1}{V_3}\int d^4z \frac{1}{|\mathbf{x}-\mathbf{z}|^2}\partial_r\big[B^{JI}\big(\partial^r K_I{}^n - \partial^n K_I{}^r + \epsilon^{rn\ell s}\partial_\ell K_{Is}\big)\big] \ , \tag{3.9}$$

where B^{JI} is the inverse of the matrix

$$B_{IJ} = e^{-2U}(2C_{IJK}X^K - 9X_IX_J) \tag{3.10}$$

and

$$K_I{}^n = -\sum_A \frac{\lambda_{IA}v_{IA}{}^n}{|\mathbf{x}-\mathbf{y}_{IA}|^2} \ . \tag{3.11}$$

It remains to calculate the moduli metric. For this, we must substitute the solutions (3.8) and (3.9) into the action including the source term and collect the terms $S^{(2)}$ and $S^{(2)}_{\text{source}}$ which are quadratic in the velocities up to surface terms. The second order term in the source free action is

$$\begin{aligned} S^{(2)} = \int d^5x \, \big[& 3e^{2U}(\partial_0 e^{2U})^2 + \frac{1}{2}e^{-6U}f_{mn}f^{mn} + \frac{3}{2}e^{6U}\partial_0 X_I \partial_0 X^I - (\partial_0 D^I{}_n)\partial^n H_I \\ &+ \frac{1}{2}e^{-2U}Q_{IJ}f^I_{mn}f^{Jmn} - \frac{3}{2}e^{-4U}X_I f^I_{mn}f^{mn} - \frac{1}{8}e^{-2U}C_{IJK}X^K\epsilon^{mnr\ell}f^I_{mn}f^J_{r\ell} \\ &+ \frac{3}{4}e^{-2U}X_I\epsilon^{mnr\ell}f^I_{mn}f_{r\ell} - \frac{1}{4}e^{-6U}\epsilon^{mnr\ell}f_{mn}f_{r\ell}\big] \end{aligned} \tag{3.12}$$

and upon substituting the solution gives

$$S^{(2)} = -\int d^5x \, \frac{1}{2}e^{4U}X^I\partial_0\partial_0 H_I + \frac{1}{8}B_{IJ}\big[f^I_{mn}f^{Jmn} + \frac{1}{2}\epsilon^{mnr\ell}f^I_{mn}f^J_{r\ell}\big] \ . \tag{3.13}$$

The second order contribution from the source terms is

$$S^{(2)}_{\text{source}} = -V_3\int d^5x \sum \delta(\mathbf{x}-\mathbf{y}_{IA})\big[X^I\lambda_{IA}|\mathbf{v}_{IA}|^2e^{4U} + 2D^I{}_m\lambda_{IA}v_{IA}{}^m\big] \ . \tag{3.14}$$

Adding (3.13) and (3.14) and using

$$
\begin{aligned}
-2\int d^5x\, V_3 \sum D^I{}_m \lambda_{IA} v_{IA}{}^m \delta(\mathbf{x}-\mathbf{y}_{IA}) \\
= \frac{1}{4}\int d^5x\, B_{IJ}\big[f^I_{mn} f^{Jmn} + \frac{1}{2}\epsilon^{mnr\ell} f^I_{mn} f^J_{r\ell}\big]
\end{aligned}
\tag{3.15}
$$

and

$$
\begin{aligned}
B^{IJ}X_J &= \frac{1}{3}e^{2U}X^I \\
B^{IJ}\partial_\mu X_J &= \frac{1}{6}e^{2U}\partial_\mu X^I \\
\partial_\mu B^{IJ} X_J &= \frac{1}{3}\partial_\mu e^{2U} X^I + \frac{1}{2}e^{2U}\partial_\mu X^I \ ,
\end{aligned}
\tag{3.16}
$$

we find that

$$
\begin{aligned}
& S^{(2)} + S^{(2)}_{\text{source}} = \\
& \int d^5x\big[-V_3\sum X^I\lambda_{IA}|\mathbf{v}_{IA}|^2 e^{4U}\delta(\mathbf{x}-\mathbf{y}_{IA}) - K_{Jn}\partial_r\big(B^{IJ}\partial^r K_I{}^n\big) \\
& \quad + B^{IJ}\big(\partial_0 H_I \partial_0 H_J - \partial_r K_{Jn}\partial^n K_I{}^r\big) + B^{IJ}\epsilon^{rn\ell s}\partial_r K_{Jn}\partial_\ell K_{Is}\big] \ .
\end{aligned}
\tag{3.17}
$$

From this we can easily read the moduli metric which turns out to be weak HKT. To see this, let (I_s) be the triplet of constant complex structures on $\mathbb{R}^4$ satisfying the algebra of the imaginary unit quaternions which are associated to self-dual 2-forms on $\mathbb{R}^4$. Then

$$
g_{mIA\ nJB} = \partial_{mIA}\partial_{nJB}\,\mu + \sum_{s=1}^{3} (I_s)^\ell{}_m (I_s)^q{}_n \partial_{\ell IA}\partial_{qJB}\,\mu \tag{3.18}
$$

where

$$
\mu = \int d^4x\, e^{6U} \tag{3.19}
$$

is the HKT potential and e^U given in (2.8); for a discussion about HKT potentials see [17, 12]. To show this, we have used

$$\sum_{s=1}^{3} (I_s)^{\ell}{}_m (I_s)^q{}_n = \delta_{mn}\delta^{\ell q} - \delta_m{}^q \delta_n{}^{\ell} - \epsilon_{mn}{}^{\ell q} \ , \tag{3.20}$$

$$\partial_{mIA}(e^{2U}) = \frac{1}{3} X^I \partial_{mIA} H_I \ , \tag{3.21}$$

(no sum over I) and

$$\partial_{mJA} X^I = -2e^{-4U} B^{IJ} \partial_m \Big(\frac{\lambda_{JA}}{|\mathbf{x} - \mathbf{y}_{JA}|^2}\Big) + \frac{2}{3} e^{-2U} X^I X^J \partial_m \Big(\frac{\lambda_{JA}}{|\mathbf{x} - \mathbf{y}_{JA}|^2}\Big) \ . \tag{3.22}$$

(no sum over I and J). For a generic choice of very special geometry and a generic choice of a black hole solution, the torsion of the HKT geometry is not a closed form. So the moduli space of the $N = 2$ supergravity black holes is a weak HKT manifold with metric (3.18) and hypercomplex structure $\{I_s\}$. We remark that the moduli metric (3.18) has a term symmetric and a term anti-symmetric in the spatial spacetime indices. The anti-symmetric piece can be written in a basis of anti-self-dual two-forms in $\mathbb{R}^4$. Such HKT geometries have been considered in the past [18].

4. Examples

One possibility is to consider black holes that are coupled to a single one-form gauge potential. For this we choose

$$C_{111} = 1 \ . \tag{4.1}$$

The corresponding action (2.4) is that of a pure five-dimensional supergravity multiplet which has bosonic fields a graviton and an one-form gauge potential [19]. We remark that our metric in this case is in agreement with that of [13].

Alternatively, we can consider the moduli space of black holes that are coupled to different one-form gauge potentials. To give an example, we shall describe in detail the metric on the moduli space of black holes of the STU model. This model arises in the context of compactifications of the heterotic string on $K_3 \times S^1$ and the associated very special geometry has been presented in [20]. In this case, we take $I, J, K = 1, 2, 3$ and the non-vanishing component of C is

$$C_{123} = 1 \ . \tag{4.2}$$

Then, X^I and e^{2U} are expressed in terms of the harmonic functions as

$$\begin{aligned} e^{2U} &= (H_1 H_2 H_3)^{\frac{1}{3}} \\ X^1 &= \big(\frac{H_2 H_3}{{H_1}^2}\big)^{\frac{1}{3}} \\ X^2 &= \big(\frac{H_1 H_3}{{H_2}^2}\big)^{\frac{1}{3}} \\ X^3 &= \big(\frac{H_1 H_2}{{H_3}^2}\big)^{\frac{1}{3}} \end{aligned} \tag{4.3}$$

Similarly, the non-vanishing components of B^{IJ} are

$$\begin{aligned} B^{12} &= \frac{1}{2} H_3 \\ B^{13} &= \frac{1}{2} H_2 \\ B^{23} &= \frac{1}{2} H_1 \ . \end{aligned} \tag{4.4}$$

Substituting these into the expression for the moduli metric, we find

$$
\begin{aligned}
ds^2 = & -V_3\big[h_2h_3\sum\lambda_{1A}|d\mathbf{y}_{1A}|^2 + h_1h_3\sum\lambda_{2A}|d\mathbf{y}_{2A}|^2 + h_1h_2\sum\lambda_{3A}|d\mathbf{y}_{3A}|^2\big] \\
& -V_3h_2\sum\frac{\lambda_{1A}\lambda_{3B}}{|\mathbf{y}_{1A}-\mathbf{y}_{3B}|^2}|d\mathbf{y}_{1A}-d\mathbf{y}_{3B}|^2 - V_3h_1\sum\frac{\lambda_{2A}\lambda_{3B}}{|\mathbf{y}_{2A}-\mathbf{y}_{3B}|^2}|d\mathbf{y}_{2A}-d\mathbf{y}_{3B}|^2 \\
& -V_3h_3\sum\frac{\lambda_{1A}\lambda_{2B}}{|\mathbf{y}_{1A}-\mathbf{y}_{2B}|^2}|d\mathbf{y}_{1A}-d\mathbf{y}_{2B}|^2 \\
& -\frac{1}{2}V_3\sum\lambda_{1A}\lambda_{2B}\lambda_{3C}|d\mathbf{y}_{1A}-d\mathbf{y}_{2B}|^2\big[\frac{1}{|\mathbf{y}_{1A}-\mathbf{y}_{3C}|^2|\mathbf{y}_{1A}-\mathbf{y}_{2B}|^2} \\
& \qquad + \frac{1}{|\mathbf{y}_{2B}-\mathbf{y}_{3C}|^2|\mathbf{y}_{2B}-\mathbf{y}_{1A}|^2} - \frac{1}{|\mathbf{y}_{1A}-\mathbf{y}_{3C}|^2|\mathbf{y}_{2B}-\mathbf{y}_{3C}|^2}\big] \\
& -\frac{1}{2}V_3\sum\lambda_{1C}\lambda_{2A}\lambda_{3B}|d\mathbf{y}_{2A}-d\mathbf{y}_{3B}|^2\big[\frac{1}{|\mathbf{y}_{2A}-\mathbf{y}_{1C}|^2|\mathbf{y}_{2A}-\mathbf{y}_{3B}|^2} \\
& \qquad + \frac{1}{|\mathbf{y}_{3B}-\mathbf{y}_{1C}|^2|\mathbf{y}_{3B}-\mathbf{y}_{2A}|^2} - \frac{1}{|\mathbf{y}_{2A}-\mathbf{y}_{1C}|^2|\mathbf{y}_{3B}-\mathbf{y}_{1C}|^2}\big] \\
& -\frac{1}{2}V_3\sum\lambda_{1A}\lambda_{2C}\lambda_{3B}|d\mathbf{y}_{1A}-d\mathbf{y}_{3B}|^2\big[\frac{1}{|\mathbf{y}_{1A}-\mathbf{y}_{2C}|^2|\mathbf{y}_{1A}-\mathbf{y}_{3B}|^2} \\
& \qquad + \frac{1}{|\mathbf{y}_{3B}-\mathbf{y}_{1A}|^2|\mathbf{y}_{3B}-\mathbf{y}_{2C}|^2} - \frac{1}{|\mathbf{y}_{1A}-\mathbf{y}_{2C}|^2|\mathbf{y}_{3B}-\mathbf{y}_{2C}|^2}\big] \\
& +2\int d^4x\sum\frac{\lambda_{1C}\lambda_{2A}\lambda_{3B}}{|\mathbf{x}-\mathbf{y}_{1C}|^2}\big(dy_{2A}{}^{[m}dy_{3B}{}^{n]}\big)^-\partial_m\big(\frac{1}{|\mathbf{x}-\mathbf{y}_{2A}|^2}\big)\partial_n\big(\frac{1}{|\mathbf{x}-\mathbf{y}_{3B}|^2}\big) \\
& +2\int d^4x\sum\frac{\lambda_{2C}\lambda_{3A}\lambda_{1B}}{|\mathbf{x}-\mathbf{y}_{2C}|^2}\big(dy_{3A}{}^{[m}dy_{1B}{}^{n]}\big)^-\partial_m\big(\frac{1}{|\mathbf{x}-\mathbf{y}_{3A}|^2}\big)\partial_n\big(\frac{1}{|\mathbf{x}-\mathbf{y}_{1B}|^2}\big) \\
& +2\int d^4x\sum\frac{\lambda_{3C}\lambda_{1A}\lambda_{2B}}{|\mathbf{x}-\mathbf{y}_{3C}|^2}\big(dy_{1A}{}^{[m}dy_{2B}{}^{n]}\big)^-\partial_m\big(\frac{1}{|\mathbf{x}-\mathbf{y}_{1A}|^2}\big)\partial_n\big(\frac{1}{|\mathbf{x}-\mathbf{y}_{2B}|^2}\big)
\end{aligned}
\tag{4.5}
$$

where

$$
\big(dy_{2A}{}^{[m}dy_{3B}{}^{n]}\big)^- = dy_{2A}{}^{[m}dy_{3B}{}^{n]} - \frac{1}{2}\epsilon^{mn}{}_{rs}dy_{2A}{}^{[r}dy_{3B}{}^{s]} \tag{4.6}
$$

and similarly for the rest. Observe that the moduli metric has the general structure of HKT metrics considered in [18].

The moduli metric indicates that there are up to three body interactions. One possible explanation for this is that these black holes are in the same universality class as the black holes that are made from *three* intersecting branes in the case of toroidal compactifications of M-theory to five dimensions. In both cases the black holes preserve the same fraction of maximal supersymmetry. So the three body interactions reflect the fact that the black holes are made from three different objects.

Now consider the case of three black holes each coupled to a different one-form gauge potential. One might have expected that the moduli metric simplifies in this case in analogy with a similar situation in the context of BPS solitons but this does not seem to be the case here. This might be due to the fact that the two scalars involved in the model have non-trivial interactions in the action.

5. Concluding Remarks

We have computed the moduli metric of five-dimensional black holes of $N = 2$ supergravity coupled to any number of vector multiplets. We have found that the moduli space is a weak HKT manifold. One can investigate the near horizon limit of our moduli metric as in [13]. In particular, we examined the near horizon limit of the STU model moduli metric that we have presented in the previous section. We found though that the moduli metric appears to be singular in this limit.

To investigate black hole scattering and black hole bound states a new way to construct these metrics may be required. Although the moduli space of n black holes can be identified with the configuration space of n particles, the computation of the metric is local. A different construction may provide the tools to find closed geodesics in the black hole moduli spaces and so investigate the presence of black hole bound states.

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REFERENCES

1. A. Strominger and C. Vafa, *Macroscopic Origin of the Bekenstein-Hawking Entropy*, Phys. Lett. **B379** (1996) 99, hep-th/9601029.
2. A.C. Cadavid, A. Ceresole, R. D'Auria and S. Ferrara,*Eleven-Dimensional Supergravity Compactified of Calabi-Yau Threefolds* , Phys. Lett. **B357** (1995) 76.
3. G. Papadopoulos and P.K. Townsend, Compactifications of D=11 Supergravity on Spaces of Exceptional Holonomy, Phys. Lett.**B357** (1995) 300, hep-th/9506150.
4. M.B. Green, J.H. Schwarz and P.C. West, *Anomaly Free Chiral Theories in Six Dimensions*, Nucl. Phys. **B 254** (1985) 327.
5. M. Günaydin, G. Sierra and P.K. Townsend, *Exceptional Supergravity Theories and the Magic Square*, Phys. Lett. **B133** (1983) 72; *The Geometry of N=2 Maxwell-Einstein Supergravity and Jordan Algebras*, Nucl. Phys. **B242** (1984) 244.
6. B. de Wit and A. van Proeyen, *Broken Sigma Model Isometries in Very Special Geometry*, Phys. Lett. **293B** (1992) 95, hep-th/9207091.
7. W.A. Sabra, *General BPS Black Holes in Five Dimensions*, Mod. Phys. Lett. **A 13** (1998) 239, hep-th/9708103; A. Chamseddine and W.A. Sabra, Phys. Lett. **B426** (1998) 36.
8. R.A.Coles and G.Papadopoulos,*The Geometry of one-dimensional supersymmetric nonlinear sigma models*, Class. Quantum Grav. **7** (1990) 427.
9. G.W.Gibbons and R.Kallosh,*Topology,entropy and the Witten index of dilaton black holes*, Phys. Rev. **D51** (1995) 2839.
10. G.W. Gibbons, G. Papadopoulos and K. Stelle, *HKT and OKT Geometries on Soliton Black Hole Moduli Spaces*, Nucl. Phys. B508 (1997) 623, hep-th/9706207.

11. P.S. Howe and G. Papadopoulos, *Twistor Spaces for HKT Manifolds*, Phys. Letts. **B379** (1996)80, hep-th/9602108.

12. G. Grantcharov and Y.S. Poon, *Geometry of hyper-Kähler Connections with Torsion*, math.dg/9908015.

13. J. Michelson and A. Strominger, *Superconformal Multi-Black Hole Quantum Mechanics*, HUTP-99/A047, hep-th/9908044.

14. R.C.Ferrell & D.M.Eardley,*Slow motion scattering and coalescence of maximally charged black holes*,Phys. Rev. Lett. **59** (1987) 1617.

15. K. Shiraishi, *Moduli Space Metric for Maximally-Charged Dilaton B;ack Holes*, Nucl. Phys. **B402** (1993) 399.

16. G.W.Gibbons & P.J.Ruback,*The motion of extreme Reissner-Nordstrom black holes in the low velocity limit*,Phys. Rev. Lett. **57** (1986) 1492.

17. J. Michelson and A. Strominger, *The geometry of (Super)conformal Quantum Mechanics*, HUTP-99/A045, hep-th/9907191.

18. G. Papadopoulos and A. Teschendorff, *Multi-angle Five-Brane Intersections*, Phys. Lett. **B443** (1998) 159, hep-th/9806191; *Grassmannians, Calibrations and Five-Brane Intersections*, hep-th/9811034.

19. E. Cremmer, *Supergravities in Five Dimensions*, in "Superspace and Supergravity", eds. S.W. Hawking and M. Roček CUP 1981, page 267.

20. I. Antoniadis, S. Ferrara and T.R. Taylor, *N=2 Heterotic Superstrings and its Dual Theory in Five Dimensions*, Nucl. Phys. **B460** (1996) 489.